Entropy generation minimization of nanofluid flow in a MHD channel considering thermal radiation effect

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Nomenclature

A - axial temperature gradient, Km⁻¹; a - half width of channel, m; B - applied magnetic field, Wb m⁻²; (C_p)h - specific heat of main fluid, J K⁻¹ kg⁻¹; (C_p)nf - specific heat of nanofluid, J K⁻¹ kg⁻¹; (C_p)nfh - specific heat of solid nanoparticle, J K⁻¹ kg⁻¹; E - electrical field, N/C; Ha - Hartman number of main fluid, (=Ba/l(μf/σf)⁰⁵); kₙf - conductivity of main fluid, W m⁻¹ K⁻¹; kₙ - conductivity of nanofluid, W m⁻¹ K⁻¹; kₙs - conductivity of solid nanoparticle, W m⁻¹ K⁻¹; k - mean absorption coefficient; P - Pressure, Pa; Pe - Peclet number, Pr Re; Pr - Prandtl number, v/α; qᵣ - radiation heat flux, (=(-Tv²σ'/3k')εT/εy); R - radiation parameter, 16σ²Tᵣ³/3k'kf; Re - Reynolds number, u₀ρf/μf; S - total entropy generation rate; Sₜ - local entropy generation; T - temperature, K; Tₛ - average temperature of the nanofluid, K; u - axial velocity component, ms⁻¹; u₀ - average velocity of the fluid, ms⁻¹; v - transversal velocity, ms⁻¹; x - axial coordinate, m; y - transversal coordinate, m;

Greek symbols –

η - electrical efficiency of power generator, E/u₀B; ϕ - nanoparticle volume fraction; μf - dynamic viscosity of main fluid, kg m⁻¹ s⁻¹; μₙ - dynamic viscosity of solid nanoparticle, kg m⁻¹ s⁻¹; μnf - dynamic viscosity of nanofluid, kg m⁻¹ s⁻¹; μs - density of main fluid, kg m⁻³; ρf - density of solid nanoparticle, kg m⁻³; ρₙ - density of nanofluid, kg m⁻³; σf - electrical conductivity of fluid, m⁻¹; σₙ - electrical conductivity of solid nanoparticle, m⁻¹; σnf - electrical conductivity of nanofluid, m⁻¹; σ - Stefan-Boltzmann constant

1. Introduction

Nowadays, the importance of the energy management from the point of view of generation and utilization are more pronounced than the old times. One of the ways of preservation of the energy resources is the optimum design of the power generators and energy conversion systems. Most recently studies have been focused on the problem of the entropy minimization in different fields of engineering, namely, in heat and mass transfer processes. Significance of entropy minimization from thermodynamics viewpoint is equal to the concept of the availability, maximization and optimal conditions in energy utilization and production. Bejan [1, 2], developed the entropy generation minimization method and introduced its applications in engineering sciences. A’Boud-Saouli et al. [3], studied the effects of viscous dissipation and magnetic field on the local entropy generation rate for laminar fluid flow through two parallel plates. They concluded that the entropy-generation increases with Hartman and Brinkman numbers. Entropy minimizations in MHD channels have been considered by some researchers as power generation tools. Ibanez et al. [4, 5], minimized the global entropy generation rate for viscous flow between two parallel plane walls of finite separation distances. They evaluated entropy generation for two simple cases of flows. They show that a minimum global entropy generation rate using asymmetric convective cooling is possible. The second low analysis of plasma flow in MHD generator was investigated by Saida and Montazeri [6]. They considered the linearly variable cross section for the MHD channel, and presented the second low efficiency and the electrical efficiency of power generation. They concluded that in generator using plasma as a flowing fluid, the influence of the ohmic dissipation is not considerable on the entropy generation and the power generation availability. They attributed this to the low conductivity of plasma compared to the liquid metal. Habibi Matin et al. [7] and Dehsara et al. [8] investigated the second law analysis of MHD flow of nanofluid over a stretching sheet in the regular and porous mediums. They showed that adding nano particles to the base fluids in forced, natural, and mixed convection would cause a reduction in shear force and a decrease in stretching sheet heat transfer coefficient. Junkowski [9], investigated the influence of the cross section of the MHD channel on the entropy generation rate. He suggested that in adiabatic flow, the circular cross section is an optimum shape for the entropy generation minimization. Chen et al. [10], numerically investigated the local entropy generation rate for mixed convection flow in a parallel vertical plates. Furthermore they used the semi-analytic method (DTM) to validate their solution. They concluded that minimum entropy generation rate occurs near the centerline of the channel. Hung [11], has taken into account the effects of viscous dissipation on
entropy generation of non-Newtonian fluids in channels. He divided the main irreversibility into two parts, heat transfer irreversibility and friction irreversibility. The total entropy generation minimization for a thermally fully developed MHD flow in a microchannel with conducting walls of finite thickness was investigated by Ibáñez and Cuevas [12]. The importance of each has been discussed. For the best of authors’ knowledge, the entropy generation minimization of a nanofluid flow in MHD power generator channel has not been investigated. The purpose of using the nanoparticles is to increase the effective electrical and thermal conductivity of the nanofluid.

In the present work, the entropy generation minimization of the nanofluid flow in MHD channel formed by two parallel isothermal plates is considered. We have considered the nanofluid as a homogenous with average physical properties of the nanoparticles and the base fluid. Considering this assumption the nanofluid from macroscopic viewpoint is similar to a single phase fluid. The main fluid is air with added Al$_2$O$_3$, Cu and Ti, nanoparticles with different volume fraction. For evaluating global entropy generation, the velocity and the temperature fields have been obtained analytically by solving the energy and momentum equations assuming fully developed flow. The total entropy generation is evaluated by integrating the local entropy generation over the whole volume of channel. Minimization of the entropy generation versus several parameters such as electrical efficiency, radiation parameter, nanoparticles volume fraction, the axial temperature gradient, Hartman and Peclet numbers have been presented and discussed.

2. Formulation of the problem

The total entropy generation minimization of nanofluid magneto-hydrodynamic (MHD) flow through a two parallel isothermal plate’s channel with thermal radiation flux included is considered. The assumption of isothermal plates for the channel is true when the thickness of the plates is very small in comparison with the height of the channel. Otherwise conduction heat losses from the plates must be incorporated. Governing fully developed momentum and energy equations assuming constant physical properties are as follow:

**Momentum:**

\[
\rho_{nf}(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}) = -\frac{\partial P}{\partial x} + \rho_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \sigma_{nf} B^2 u \quad (1);
\]

**Energy:**

\[
(\rho C_p)_{nf} \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \sigma_{nf} B^2 u^2 + \mu_{nf} \left( \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} \right) - \frac{\partial q_t}{\partial x} - \frac{\partial q_l}{\partial y} \quad (2),
\]

where \( u \) and \( v \) are the velocity components in \( x \) and \( y \) directions respectively as shown in Fig. 1. \( P \) is the pressure. \( T \) is the temperature field; \( \rho_{nf}, \mu_{nf}, \sigma_{nf}, k_{nf}, \) and \( (\rho C_p)_{nf} \) are effective density, effective dynamic viscosity, effective electric conductivity, effective thermal conductivity and effective heat capacitance of the nanofluid, respectively and defined as following [13, 14]:

\[
\rho_{nf} = (1-\phi) \rho_f + \phi \rho_p \quad ;
\]

\[
\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2/3}} \quad ;
\]

\[
\sigma_{nf} = (1+3\phi) \sigma_f \quad ;
\]

\[
k_{nf} = \frac{k_f + 2k_j - 2\phi(k_j - k_f)}{k_i + 2k_j + 2\phi(k_j - k_f)} \quad ;
\]

\[
\left(\rho C_p\right)_{nf} = (1-\phi)\left(\rho C_p\right)_f + \phi(\rho C_p)_p \quad ,
\]

where \( \phi \) is defined as nanoparticles volume fraction.

Subscripts \( s \) and \( f \) denote nanoparticles and the main fluid properties respectively. \( B \) is a transverse magnetic field that we assume to be applied in the \( x \) direction and \( q_t \) is the thermal radiation flux. We assume that the flow is hydro-dynamically and thermally fully developed in the \( x \) direction that is \( v = 0, \frac{\partial u}{\partial x} = 0, \frac{\partial^2 T}{\partial x^2} = 0 \), and \( \frac{\partial q_t}{\partial x} = 0 \). Therefore the momentum and energy equations can be rewritten as follow:

\[
- \frac{\partial P}{\partial x} + \frac{\mu_f}{(1-\phi)^{2/3}} \frac{\partial^2 u}{\partial y^2} (1+3\phi) \sigma_f B^2 u = 0 \quad ; (8)
\]

\[
\left(1-\phi\right)\left(\rho C_p\right)_f + \phi(\rho C_p)_p \frac{\partial T}{\partial x} = \frac{k_f + 2k_j - 2\phi(k_j - k_f)}{k_i + 2k_j + 2\phi(k_j - k_f)} \frac{\partial^2 T}{\partial y^2} + (1+3\phi) \sigma_f B^2 \left(u - \eta \right)^2 + \frac{\mu_f}{(1-\phi)^{2/3}} \left( \frac{\partial u}{\partial y} \right)^2 - \frac{\partial q_t}{\partial y} \quad , (9)
\]

with the following boundary conditions:

\[
u \left( y = 0 \right) = 0 \quad ; (10)
\]

\[
\frac{\partial u}{\partial y} \left( y = 0 \right) = 0 \quad ; (11)
\]

\[
T \left( y = a \right) = T_i \quad ; (12)
\]

\[
T \left( y = -a \right) = T_s \quad . (13)
\]

The effective velocity of the nanofluid through channel is due to two elements, the velocity of the inlet flow and the influence of the Lorentz force. The interaction between the magnetic field and the electrically conducting fluid flow produces a resistive force against the fluid flowing known as Lorentz force. By definition \( \eta = E/u_0B \), the electrical efficiency of the MHD power generator [6], where \( E \) is delivered electric field and \( u_0 \) is the mean velocity of the fluid in the cross section of the channel. The temperature difference within the flow are assumed to be sufficiently small such that \( T^* \) may be expressed as a linear function of temperature, i.e., \( T^* \equiv 4T_i^2T - 3T_i^4 \).
fully developed in the x direction that is, $\epsilon \frac{\partial T}{\partial x} = A$, where $A$ is the axial temperature gradient and assumed to be constant Snyder [15]. By integrating Eqs. (16) and (17) along with the boundary conditions, as mentioned in relations through (18) to (21), the following velocity and temperature profiles are obtained:

$$u = \frac{Ha_0 [\cosh (Ha_0) - \cosh (Ha_0 Y)]}{Ha_0 \cosh (Ha_0) - \sinh (Ha_0)};$$  \hspace{1cm} (22)

$$T = AX - \frac{2\alpha_1}{Ha_0^2} \cosh (Ha_0 Y) - \frac{\alpha_2}{2Ha_0^2} \cosh (2Ha_0 Y) + \frac{\alpha_3}{2} Y^2 + \alpha_4 Y + \alpha_5,$$  \hspace{1cm} (23)

wherein, the coefficients are as follows:

$$\begin{align*}
\alpha_1 &= C_3 + (Ha_0)^2 (C_1)^2 (\eta' - \cosh (Ha_0)) \frac{1}{1 + R} \\
\alpha_2 &= \frac{(Ha_0)^2 (C_1)^2}{4(1 + R)} + C_4; \\
\alpha_3 &= C_2 - (Ha_0)^2 (C_1)^2 \eta' - \cosh (Ha_0) \frac{1}{1 + R} \\
\alpha_4 &= \frac{\bar{T} - T}{2}; \\
\alpha_5 &= \frac{\bar{T} + T}{2} + \frac{2\alpha_1}{Ha_0^2} \cosh (Ha_0) + \\
&\quad + \frac{\alpha_2}{2Ha_0^2} \cosh (2Ha_0) + \frac{\alpha_3}{2}.
\end{align*}$$  \hspace{1cm} (24)

In the above relations parameters are defined as:

$$\begin{align*}
Ha_0 &= Ha \sqrt{(1 + 3\phi)(1 - \phi)}^2; \\
C_1 &= \frac{Ha_0}{Ha_0 \cosh (Ha_0) - \sinh (Ha_0)}; \\
C_2 &= \frac{A Pe C_1 \cosh (Ha_0)}{1 + R}; \\
C_3 &= \frac{A Pe C_1}{2(1 + R)}; \\
C_4 &= \frac{(Ha_0)^2 (C_1)^2}{4(1 + R)}; \\
\eta' &= \frac{\eta}{C_1}.
\end{align*}$$  \hspace{1cm} (25)

The properties of the nanoparticles and the basic fluid used in this investigation at $T = 1000$ Kelvin are given in Table. Radiation parameter $(R)$, $Ha$ and $Pe$ numbers, are defined as follows:

$$Ha = \frac{B a}{\sqrt{\frac{\mu_f}{\sigma_f}}}; \quad Pe = Pr Re = \frac{u_0 a (\rho C_p)}{k_f}; \quad R = \frac{16\sigma^* T_0^3}{3M_k k_f}. \hspace{1cm} (26)$$
3. Entropy generation

The local entropy generation rate produced in channel has four different sources, heat flow, ohmic dissipation and viscous dissipation. The local entropy generation can be written as Groot and Mazur [16]:

$$ S = - \left[ \left( \frac{\partial T}{\partial X} \right)^2 + \left( \frac{\partial \theta}{\partial Y} \right)^2 \right] + \frac{\partial^2 \theta}{\partial Y^2} + \frac{H_{\theta_0}}{T} \left( \frac{u - \eta}{T} \right)^2. \quad (27) $$

The first term on the right hand side of the above equation represents the entropy generation produced by heat flow, the second term suggests the entropy generation due to viscous dissipation and the last terms account for ohmic dissipation. To evaluate the total entropy generation rate it is necessary to integrate the local entropy generation rate $S$, over a unit volume of the channel. We use the Simpson numerical technique for integrating over a unit volume. The total entropy generation rate $S$ is obtained versus non-dimensional parameters such as $Ha$, $\eta$, $R$, $Pe$, $\phi$, $A$ and temperatures of the upper and the lower walls of the channel. Although a vast range of the above parameters can be selected for minimization, our minimization is accomplished for arbitrary values of governing parameters. Entropy is minimized respect to one parameter whereas other parameters are kept constant.

4. Results and discussion

In the present paper we focus on the entropy generation minimization of nanofluid MHD flow in channel. The minimum entropy conditions provide possibility of reaching to the maximum available work or in the other word increases the exergy of the power generation systems. We attempt to find the optimum conditions for MHD channel power generator versus governing physical parameters such as $Ha$, $\eta$, $R$, $Pe$, $\phi$, $A$ and temperatures of the upper and the lower walls. Fig. 2 shows the effect of $A$, axial temperature gradient on the total entropy generation rate with three values of electrical efficiency. As the axial temperature gradient increases, first, the entropy generation decreases and approaches the minimum value near axial temperature gradient $A \approx 60$ and then increases. For $\eta = 2$, the minimum entropy generation shift to upper value of the axial temperature gradient. In Fig. 3 the total entropy generation rate is plotted versus radiation parameter in the presence of three values of nanoparticles volume fraction. As it can be seen in this figure, when the volume fraction is $\phi = 0$ for $R \approx 1$ the total entropy generation rate is at the minimum value while when the volume fraction increases value of radiation parameter in which entropy generation rate is minimized increases. Fig. 4 shows the total entropy generation versus temperature of the bottom wall of the channel at three different values of the upper wall temperatures. From this figure, it is clear that for each value of the upper wall temperature, there is a minimum value for total entropy generation. As the temperature of the lower wall increases, this minimum value tends to happen at higher temperature.

Furthermore, the minimum value of the total entropy generation when the upper and the bottom walls are at the same temperature get closer to the corresponding temperature under consideration. The total entropy generation
has been plotted in Fig. 5 for various values of Pe and radiation parameter. As it is observed from this figure, there is an optimum value for entropy generation for each particular Pe. The interesting point which should be mentioned is that, at higher values of R (radiation parameter), the minimum value of entropy generation occurs at higher values of Pe. Fig. 6 shows the nanoparticles volume fraction effect on the total entropy generation rate for three values of Ha. As it can be seen from this figure, the optimum value of 0.2% volume fraction of the nanoparticles added would minimizes the total entropy generation rate and when Ha increases value of volume fraction in which entropy generation rate is minimized increases.

Fig. 7 shows the influence of the Ha, on the total entropy generation rate is minimized increases.

![Fig. 5 Total entropy generation versus Pe for different values of radiation parameter at Ha = 0.15, η = 5, φ = 0, A = 90, Pe = 7, T1 = 300, T2 = 1000](image)

![Fig. 6 Total entropy generation versus Ha for different values of nanoparticles (Al2O3) volume fraction at η = 1, A = 10, R = 1, Pe = 3, T1 = 300, T2 = 1500](image)

![Fig. 7 Total entropy generation versus Ha for different types of nanoparticles at η = 2, A = 20, φ = 0.001, R = 1, Pe = 10, T1 = 300, T2 = 1000](image)

### Table

<table>
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<tr>
<th>Physical properties</th>
<th>Fluid phase (air)</th>
<th>Ti</th>
<th>Al2O3</th>
<th>Cu</th>
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<td>-</td>
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</tbody>
</table>

Thermo-physical properties of air and nanoparticles at 1000 K.

### References


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NANOSKYSČIO SRAUTO ENTROPIJOS GENERAVIMO MINIZIMAVIMAS MHD KANALE
ATSIZVĖLGIANT Į ŠILUMOS SKLIDIMO POVEIKĮ

Reziumė


M. Habibi Matin, R. Hosseini, M. Simiari, P. Jahangiri

ENTROPY GENERATION MINIMIZATION OF NANOFLOUID FLOW IN A MHD CHANNEL
CONSIDERING THERMAL RADIATION EFFECT

Summary

The purpose of this article is the total entropy generation minimization of the nanofluid magnetohydrodynamic (MHD) flow through a channel formed by two parallel isothermal plates considering thermal radiation effect. Air is the main fluid and three types of nanoparticles such as Al2O3, Ti and Cu are examined. The velocity and temperature fields are obtained from solution of the momentum and energy equations analytically. The local and total entropy generation rates are calculated and then the total entropy generation rate is presented graphically versus governing dimensionless parameters. Electrical efficiency, radiation parameter, nanoparticles volume fraction, Hartman and Peclet numbers are considered in this work. The results show that there are conditions in which the total entropy generation rate could be optimized.

Keywords: Entropy generation; channel; Minimization; Nanofluid; MHD.

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