Simulation of vibratory alignment of the parts to be assembled under passive compliance

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1. Introduction

During the automated assembly task, when parts are fed into an assembly station, there may be the significant errors of the interdependent position and orientation. Assembly operation can be difficult and complex when mating parts are not perfectly aligned. This may result the unsuccessful assembly and, in some cases, the damage of the parts to be assembled. In order to ensure the high efficiency of the automatic assembly equipment and successful mating during the assembly operation, the efforts have been made in the area of the parts alignment research and development of the assembly devices, as well as various alignment and insertion strategies. The motion of the compliantly supported part required to compensate the positional and angular errors is known as the alignment motion, which is performed with either passive or active compliance or as the combination of the both. Passive device contains no source of energy and, therefore, comprises the elastic elements. The interaction forces of the being assembled parts are directly influencing the trajectory of the gripper during the assembly operation and thus the part-to-part position error is corrected. In active devices the sensors are used, which are fitted into the closed loop feedback system to gauge the contact information and, based on this strategy, to minimise any error that may emerge. Therefore, this approach is more expensive and has a relatively slow response.

The economic interest of the automation of assembly operation is to increase the productivity and operational reliability, to reduce unit assembly costs. One of the effective solutions of this problem is to put in practice the vibratory assembly method together with the passive compliance device. The combination between the vibration technique and the passive compliance is the best solution for the application in case of the chamferless assembly.

The passive compliance mechanism use imminent or designed flexibility to adjust the compliantly supported part’s position or orientation to perform the assembly tasks. The advantages of such mechanisms include the fast response to external constraints, as well as the sensorless control. Furthermore, this technique can also be used for the chamfered parts assembly. Most of the specific passive compliance devices have to be designed and manufactured for the parts with different geometries. However, the compliance device generally is such a mating device, which has been proposed to aid the alignment and insertion during the assembly operation. The best known passive compliance device is the remote center of compliance (RCC) [1, 2]. The arrangement of the elastic elements defines a point in the space in such a way that the applied forces produce displacements and the applied torques produce rotations about this point.

There are many research works done using the passive compliance device for assembly or other tasks. In the works [3, 4] the geometric and dynamic analysis of the compliantly supported peg-in-chamfered-hole assembly is presented. Mentioned scientific work considers the different factors, such as inertia forces, the loadings of the compliance device, the location of compliance centre and the insertion speed influence on the dynamic insertion process. An amount of numerical experiments have been implemented. It is determined that the lateral and angular compliance has a decisive influence on the reduction of the insertion force and avoiding the wedging and jamming. The assembly method for chamferless parts is proposed which combines the vibration techniques and passive compliance [3]. Vibrations are provided to the work table along the horizontal plane and thus the search of the parts alignment is performed. It should be noted, that, applying the vibrations along the horizontal plane, the process of the alignment has a random character and thus is not stable.

The study of the geometrical and dynamical conditions for the successful assembly operation of the chamferless parts is presented in the work [5]. The analysis is based both on the quasi-static peg-in-hole insertion model under relatively low insertion speed and on the approach of the passive accommodation. However, the emergence of the forces which result the peg motion during the chamferless parts’ alignment is not explored in this paper. The motion behaviour of a polygonal peg during the insertion into the chamfered hole is presented in the paper [6]. The dependences between the geometric dimensions of a polygonal peg, the exerted torque and the tilt angle are defined, which ensure the successful assembly. It is determined that the maximal allowable misalignment error, when successful assembly is possible, diminishes as the number of the peg sides increases.

The passive compliance devices can compensate the axial and angular misalignments due to elastic deformation resulted by the contact forces between the parts to be assembled. In order to provide the successful compensation of the misalignments, at least one of the parts must be chamfered and the first contact between the parts has to be within the chamfer area. In the case of the chamferless parts, the axial misalignment has to be smaller than the clearance between them. These conditions are limiting the passive compliance device application for the chamferless parts assembly.

The effective vibrational assembly method, when
passive compliance is used for the assembly of chamfered and chamferless parts, was proposed [7, 8]. This method is based on the motion of the compliant part due to a vibratory excitation, which is provided to at least one of the parts. The compliance was achieved through the use of the elastic elements, i.e. so called bellows (sylphons). When one of the parts is excited by vibrations of particular frequency and amplitude, the phenomenon of the dynamic directionality is taking place. Thus, the compliant part can displace and turn in respect to the immovable part and in such a way the linear and angular interdependent position errors of the parts to be assembled are eliminated. For compliant device with bellows the remote centre of compliance or elastic centre is not characteristic. Thus, the lateral and angular errors cannot be absorbed independently.

Currently we are making the efforts on the research of the vibratory assembly using the passive compliance device with RCC. In the paper [9] the experimental analysis of the peg-in-hole alignment and insertion is presented, when the peg is located in the compliance device of unvarying stiffness. The experiments have been carried out providing the excitation to the immovable bushing located on the platform of the vibrator. For the automated assembly it is possible to use the passive compliance devices, which have the possibility to change the parameters of the elastic elements. Such devices are able to change the elastic properties and are self-adjusting to the changing operational conditions in real time. In the assembly systems generally the stiffness of the elastic elements is changed or the composition materials with varying mechanical characteristics are used [10, 11].

This paper aims to simulate the vibratory alignment of the peg supported by compliant devices in respect of the bushing so as to establish the characteristic of the motion of the compliant peg. The dynamic model and mathematical approach of the peg alignment are developed based on the geometric constraints and deformable characteristic of the RCC device. The influence of the parameters of the mechanical system and vibratory excitation on the process of alignments is defined.

2. The model and equations of the vibratory alignment of the compliant part

The compliant peg motion during the alignment in respect of the vibratory excited bushing can be represented by a planar model. The dynamical model comprises the attached to the remote center compliance device peg and contacting with it bushing (Fig. 1). The compliant peg is presented by a mass \( m \) whereas the immovably fixed bushing is presented by a supporting base with a slot. The coordinate system \( XOZ \) is related to the edge of the supporting base slot. The remote center compliance device is represented by a gripper and elastic elements \( K_N, K_A, K_P \), which are restraining the motion of the body along the plane \( XOZ \).

The position of the body, in respect of the support, is defined by the coordinates \( X_s, Z_s \) of the mass center of the remote center compliance device and turn angle \( \varphi \) about the support point \( O \). While using the RCC device, the point \( O \) also is the remote compliance center, because, during the alignment, the peg is able to tilt in respect of this point. The parameter \( \Delta \) indicates the axial misalignment of the parts. The immovable system of the coordinates is denoted as \( X_0O_1Z_1 \).

Due to the part-to-part location errors in assembly position their axes and also the contours of the connecting surfaces do not coincide. The error vector of the axial misalignment between the mating parts may point in different directions, depending on the direction of the positioning error of the parts in assembly position. To accomplish the vibratory positioning, the peg should be pressed towards the bushing by the particular force. Due to a pressing force the peg slightly tilts in respect of the bushing and so the force asymmetry of the mechanical system occurs. Due to the vibratory excitation of the bushing, the compliant peg moves in a vertical plane towards the axial alignment of the parts.

\[
\begin{align*}
mx_G^* + H_x X_G' + K_x (X_G - X_0) &= N (\sin \varphi \mp \mu \cos \varphi); \\
mg + H_z Z_G' + (Z_G - Z_0) &= -mz_G^* + N (\cos \varphi \mp \mu \sin \varphi) - mg; \\
I (Q) \varphi'' + H_x \varphi' + K_x (\varphi - \varphi_0) &= -K_x (X_G - X_0) L_c \cos \varphi + K_z (A_1 \sin \omega t + Z_0) \left[ X_G + (L_c - L_0) \sin \varphi \right] + \\
& + mgX_G + N (Q + \mu L_c),
\end{align*}
\]

where \( H_x, H_z \) and \( H_x \) are damping coefficients; \( X_0, Z_0, \) and \( \phi_0 \) are the coordinates of the static position of the body; \( N \) is the normal force at the point of contact; \( \mu \) is dry friction coefficient at the contact point of the body and the supporting base; \( X_G^*, X_G^*, Z_G^*, Z_G^*, \varphi^*, \varphi' \) are the first and second derivatives of the corresponding generalized coordinates;

![Fig. 1 Dynamic model of the peg-to-hole alignment](image-url)
If the body is moving towards the axial alignment, the force arm $Q$ of the normal reaction $N$ at the point of contact increases. The coordinate $X_G$ increases and also changes the rotation angle $\varphi$ of the body. The prerequisite for the matching of the connective surfaces, as the body is easily inserted into the slot, is expressed as follows:

$$(X_G - L_s \sin \varphi)/\cos \varphi \geq L \cos \varphi.$$ (3)

During the motion also changes the inertia moment $I(Q)$ of the masses of the device and the body in respect of the contact point, because, as changes the parameter $Q$, the mass center - contact point distance also changes. The inertia moment is calculated by the dependence:

$$I(Q) = mL_c^2 = mL_c^2 + Q^2.$$ (4)

To obtain the generalized results of simulation, the dimensionless parameters are introduced:

$$\tau = p t; \quad \bullet = d/d\tau; \quad h = \sqrt{K_Z/m}; \quad x_0 = X_G/L; \quad z_0 = Z_G/L; \quad v = \alpha/ho; \quad h_s = H_s/\sqrt{K_Z/m}; \quad h_l = H_l/\sqrt{K_Z/m}; \quad k_s = K_s/K_z; \quad k_l = K_l/K_z; \quad r_p = (l_s^2 + q^2)/l^2; \quad n = N/(LK_z); \quad a_c = A_c/L; \quad d = g/(p^2 L); \quad l_c = L_c/L; \quad \delta = \Delta/L; q = Q/L; l_o = L_o/L.$$

Then the equations of motion may be written in a dimensionless form:

$$Q = (X_G - L_s \sin \varphi)/\cos \varphi.$$ (5)

The condition for the body insertion into the slot in dimensionless form is:

$$(x_0 - L_s \sin \varphi)/\cos \varphi \geq l \cos \varphi.$$ (6)

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Then the equations of motion may be written in a dimensionless form:

$$Q = (X_G - L_s \sin \varphi)/\cos \varphi.$$ (8)

The condition for the body insertion into the slot in dimensionless form is:

$$Q = (X_G - L_s \sin \varphi)/\cos \varphi \geq l \cos \varphi.$$ (9)

By entering the expression of the normal force into the first and third equations of the system (5), the motion equations of the body contacting the supporting base are obtained:

$$\begin{aligned}
\dot{x}_0 + h_s \dot{x}_0 + k_s x_0 = (d - a_c \nu^2 \sin \nu \tau - z_0)(\sin \varphi \mu \cos \varphi)/(\cos \varphi \pm \mu \sin \varphi) + k_s (l - \delta + l_c \sin \varphi); \\
\dot{r}_p \dot{\varphi} + h_s \dot{\varphi} + k_s \dot{\varphi} = -k_s (x_0 - x_c) \; l_c \cos \varphi + (a_c \sin \nu \tau + z_0) [x_0 + (l_c - l_o) \sin \varphi] + dx_c + k_s \varphi_0 + (d - a_c \nu^2 \sin \nu \tau - z_0) \times \dot{\varphi} / (\cos \varphi \pm \mu \sin \varphi).
\end{aligned}$$ (10)

3. The simulation of the compliant peg alignment

Taking into account the made mathematical model of the parts alignment, the applied MATLAB simulation program was written. The equations of the compliant part motion (Eq. (10)) are solved numerically and time dependences of the mass center displacement $x_0$ of the body tilt $\varphi$ of the body and also the dependences of the velocities $\dot{x}_0$ always contacting the supporting base. If changes the turn angle $\varphi$ of the moving body, the coordinate $z_0$ of the mass center can slightly change. Because the angle $\varphi$ is relatively small, it is possible to neglect the change of the coordinate $z_0$. Therefore, using the second expression in the Eq. (5), it is possible to calculate the normal reaction:

$$n = (d - a_c \sin \nu \tau - z_0)/(\cos \varphi \pm \mu \sin \varphi).$$ (11)

By entering the expression of the normal force into the first and third equations of the system (5), the motion equations of the body contacting the supporting base are obtained:

$$\begin{aligned}
\dot{x}_0 + h_s \dot{x}_0 + k_s x_0 = (d - a_c \nu^2 \sin \nu \tau - z_0)(\sin \varphi \mu \cos \varphi)/(\cos \varphi \pm \mu \sin \varphi) + k_s (l - \delta + l_c \sin \varphi); \\
\dot{r}_p \dot{\varphi} + h_s \dot{\varphi} + k_s \dot{\varphi} = -k_s (x_0 - x_c) \; l_c \cos \varphi + (a_c \sin \nu \tau + z_0) [x_0 + (l_c - l_o) \sin \varphi] + dx_c + k_s \varphi_0 + (d - a_c \nu^2 \sin \nu \tau - z_0) \times \dot{\varphi} / (\cos \varphi \pm \mu \sin \varphi).
\end{aligned}$$ (12)
and φ are obtained, which provide the possibility to estimate the character of the alignment process of the compliant part. Initially, due to a robot positioning errors the axis of the RCC device is displaced in respect of the axis of the bushing’s hole by a distance Δ. Therefore, the axial misalignment of the body in respect of the axis of the support slot is defined by the dimensionless parameter δ. The approached to the support body contacts the lateral edge of the slot and being pressed towards the support tilts to a particular angle φh, which is dependent on the pressing force. Exciting the support, the body, contacting the support, moves towards the matching of the connective surfaces. By the graphical dependences obtained during the numerical simulation under particular parameters of the dynamic system and excitation, it was defined the parameters influence on the alignment process. The alignment duration dependences on different parameters have been made, which characterise not only the intensity of the alignment, but also indicate the magnitudes of the parameters, which ensure the successful matching of the connective surfaces.

The force of the body pressing to the support, which is characterized by the parameter z0, has a significant influence on the alignment process. Under the action of this force, the body tilts slightly in respect of the support and therefore, emerges the horizontal component of the normal force, which is pointed in the displacement direction. Furthermore, under acting pressing force, the elastic elements of the RCC device get deformed and result the additional elastic force, which stipulates the axial alignment of the body. The vibratory displacement of the non-linear system may occur only under existing asymmetry of it. Even a negligible tilt of a body results a force asymmetry of the mechanical system. When the bushing, which is contacting the tilted body, is excited along the axial direction, the kinematically asymmetry of the system emerges.

The character of the alignment duration dependences on the pressing force is near linear. The alignment duration is also dependent on the stiffness coefficient kx of the elastic elements of the device along the axis X (Fig. 2).

![Fig. 2](image)

Fig. 2 The alignment duration τ dependences on the pressing force z0, under different stiffness coefficient kx.

The stiffness kx predetermines the magnitude range of the parameter z0, wherein the alignment is successful. When kx = 0.9 the alignment takes place only up to z0 = 0.5 magnitude and duration is minimal, if compared to that under different magnitudes of kx. The pressing force also predetermines the normal force at the point of contact. Furthermore, the friction force, acting against the displacement of the body, is also dependent on the normal force. The normal force, acting during the alignment, consists of the constant pressing force and of a variable component. The variable component is resulted by vibratory excitation of the bushing along the axial direction. The increasing pressing force causes the increase in friction between the contacting parts, therefore the alignment lasts longer.

It is important to determine how the amplitude and frequency of vibratory excitation are influencing the process of alignment. The amplitude of excitation may be varied within a rather broad range. An increase in excitation amplitude results almost uniform increase in alignment duration (Fig. 3). The influence of the amplitude both on the duration of the alignment and successful matching of the connective surfaces is also dependent on the other parameters. For example, under higher magnitudes of kx, the process of the alignment is more intensive. But under kx < 0.2, the alignment of the parts is not taking place.

![Fig. 3](image)

Fig. 3 The alignment duration τ dependences on the excitation amplitude a1 under different stiffness kx.

It should be noted, that using the remote center compliance device it is possible to observe the particular influence of the stiffness coefficient kx on the alignment duration (Fig. 4). The higher the magnitude of kx, the more rapidly the alignment goes. Due to the acting pressing force the elastic elements of the device get deformed, the peg tilts towards the axis of the bushing and so emerges the elastic force, which is pointed towards the alignment of the peg. This force is directly proportional to the stiffness coefficient kx.

![Fig. 4](image)

Fig. 4 The alignment duration τ dependences on the stiffness coefficient kx under different pressing force z0.

For the successful alignment of the parts it is necessary to adjust the magnitudes of the pressing force z0 and
the coefficient $k_c$. By increasing the pressing force, the alignment of the parts takes place within a range of smaller magnitudes of $k_c$ (Fig. 4).

Fig. 5 illustrates both the excitation frequency $\nu$ and excitation amplitude $a_1$ influence on the duration and success of the alignment. Within a range of relatively small amplitudes $a_1 = (0.1...0.3)$, the increase in excitation frequency results only marginal increase in alignment duration.

While increasing the excitation amplitude, the range of excitation frequencies, wherein the alignment is successful, gets narrower. Under excitation frequency $\nu = 0.6$, the duration of the alignment is marginally dependent on the amplitude. Within the range of higher amplitudes $a_1 = (0.5...0.9)$, as excitation frequency increases, the sharply slower process of the alignment is observed. Furthermore, within this range of the amplitudes, if smaller magnitudes are chosen, the successful alignment occurs within a more wide range of range of frequencies.

![Fig. 5 The alignment duration $\tau$ versus the excitation frequency $\nu$, under different excitation amplitudes $a_1$](image)

Parameter $k_a$ characterizes the angular stiffness of the remote center compliance device in respect to the contact point of the body. The magnitude of this coefficient depends on the construction of the device and on location of the elastic elements. The alignment is successful within a wide range of the stiffness $k_a$, but duration of the alignment changes nonuniformly (Fig. 6). The character of the duration dependence changes slightly depending on the excitation amplitude. The duration of the alignment is minimal when $k_a$ is within a range 35..45.

![Fig. 6 The alignment duration $\tau$ dependences on the stiffness $k_a$, under different excitation amplitudes $a_1$](image)

Both the excitation frequency and angular stiffness influence on the alignment duration is presented by the curves of the Fig. 7, which are made under excitation amplitude $a_1 = 0.3$. The character of the dependences is similar, but smaller stiffness results the smaller duration of the alignment. Independent on the magnitude of $k_a$ the alignment lasts longer under higher frequency of excitation.

![Fig. 7 The alignment duration $\tau$ dependences on the excitation frequency $\nu$, under different angular stiffness $k_a$](image)

The coefficient $\mu$ of the dry friction between the contacting parts has a significant influence on the character of the parts alignment (Fig. 8). The magnitude of this coefficient depends both on the material of the parts and roughness of the contacting surfaces. Furthermore, the coefficient of friction predetermines the friction force which resists the displacement of the peg during the alignment. Simulation results show, that under $\mu = 0.05...0.09$, the alignment of the body in respect of the slot is not occurring. When $\mu = 0.01...0.05$, the character of the displacement of the body and also the duration of the alignment are not dependent on the excitation amplitude. Minimal duration of the alignment is obtained when $\mu = 0.10$.

The increase in the friction coefficient up to $\mu = 0.15$, causes a rapid increase in alignment duration, which later is nonsignificantly dependent on $\mu$.

![Fig. 8 The alignment duration dependences on the friction coefficient $\mu$, at different excitation amplitudes $a_1$](image)

The parameter $l_G$ characterizes the mass center position of the fixed peg, whereas parameter $l_C$ indicates the position of the elastic elements of the device in respect of the contact point. The values of mentioned parameters depend on the construction of the device and also on the dimensions of the immovably fixed part. When $l_G$ changes
within a range 0.1…0.35, the duration of the alignment sharply decreases, whereas the further increase in this parameter results the nonsignificant change in duration \( \tau \) (Fig. 9). The parameter \( l_c \) has a little influence on the duration of the alignment (Fig. 10).

![Fig. 9 The alignment duration versus parameter \( l_c \) dependences under different excitation frequency](image)

![Fig. 10 The alignment duration dependences on the parameter \( l_c \) under different pressing force \( z_0 \)](image)

While analysing the parameters influence on the duration of the alignment, some parameters were varied within a particular range, while the other parameters were kept constant.

During the simulation the following values of the constant parameters have been used: \( z_0 = 0.5; \; \nu = 0.5; \; \delta = 0.3; \; k_c = 0.6; \; k_p = 60; \; a_t = 0.3; \; d = 1.5; \; \mu = 0.1; \; l_c = 1.0; \; h_t = 0.05; \; h_p = 0.5 \).

\[ l_c = l_c(\nu) \]

\[ \nu = 0.9 \]

\[ \nu = 0.1 \]

\[ \nu = 1.3 \]

\[ z_0 = 0.8 \]

\[ z_0 = 0.6 \]

\[ z_0 = 0.3 \]

\[ z_0 = 0.1 \]

\[ \tau = \tau(l_c) \]

\[ \tau = 1.90 \]

\[ \tau = 2.10 \]

\[ \tau = 2.30 \]

\[ \tau = 2.70 \]

4. Conclusions

The dynamic model of the contacting parts interaction during the assembly and the mathematical approach of the compliant peg-in-hole alignment are proposed. The numerical simulation of the peg vibratory alignment, when the bush is excited in the axial direction, is carried out. The peculiarities of the alignment process and different parameters influence on the motion of the peg are analysed varying the parameters of the dynamic system and excitation.

The successful alignment is possible only providing the predetermined pressing force between the parts being aligned. It is determined that an increase in pressing force, amplitude and frequency of excitation yields a growth in the alignment duration. To ensure the successful alignment under increasing excitation frequency it is necessary to decrease the amplitude of vibrations. The wide range of the values relevant to the angular stiffness exists, wherein the successful alignment occurs. It is possible to determine such a stiffness which results the minimal duration of the alignment. The coefficient of the sliding friction influences the motion of the peg during the alignment. To ensure the successful alignment, it is necessary to adjust the pressing force and parameters of the excitation, considering the friction coefficient between the parts being assembled.

References

SUMMARY

The paper aims to simulate the process of the compliantly supported peg alignment with respect to the bushing, which is provided with vibrations in the axial direction and to analyse the influence of the parameters of the dynamic system and excitation on the alignment process. Considering the lateral and angular compliance of the supported device the mathematical model of the parts interaction during the alignment process is formed and numerical simulation is performed. The dependencies of the alignment duration upon the pressing force of the parts, the lateral and angular stiffness of the elastic constraints, the amplitude and frequency of the excitation, the coefficient of sliding friction and geometrical parameters of the compliant device are determined. The alignment can be performed only when the parameters of the mechanical systems and excitation are adjusted. The alignment of the peg occurs during the motion from the static to the dynamic position of equilibrium of the mechanical system. The distance between these positions determines the maximum axial misalignment of the parts under which the alignment is still possible.

Keywords: automated assembly, vibration, alignment, passive compliance.

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