Model and experimental verification on actuator of magnetically controlled shape memory alloy

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1. Introduction

Magnetically controlled shape memory alloy (MSMA) is a new functional material which is found in recent years. The material can produce large induced strain by magnetic field, and has fast dynamic response, high efficiency of electromagnetic-mechanical conversion and excellent controllability. High strains of 10-15% for MSMA materials with different structure are achieved [1, 2]. The complete field-induced strain can be obtained in 250 μs [3]. Usual applications of the MSMA are actuators that produce linear motion [4-6]. Currently, MSMA has been successfully actuated at frequencies well above 1 kHz.

To realize the applications of the magnetically controlled shape memory alloy needs to foresee the MSMA output in different external conditions. The physical modeling contributes to research magnetically controlled shape memory effects of the MSMA material. Although the research model obtained a better experimental verification at present, but these models only are established basing on the deformation mechanism of MSMA material itself, there isn’t the completely description of mathematical model for the external characteristic of MSMA actuators. In order to guide the design, control and application of the actuator, it is necessary to develop the research work of the actuator model.

On the basis of thermodynamic models and the computational method of magnetic field-induced stress, considering the influence of the magnetic field, pre-pressure and temperature, the models of output strain and force are established for the MSMA actuator in this paper. The model provides theoretical foundation for the design and engineering application of the MSMA actuator.

2. Deformation mechanism, modeling theory and equivalent model of MSMA element

2.1. Deformation mechanism

The operation principle of MSMA actuators is based on the shape variation of the MSMA materials under external magnetic field. Fig. 1 shows an illustration of the reorientation principle of the twin structure by applied magnetic field [4, 5]. In the absence of the external magnetic field, magnetization vectors lie along directions of easy magnetization. This situation is shown in Fig. 1, a. Magnetization is aligned parallel to one side of the unit cell in each variant. When an external magnetic field is applied, the magnetization vectors tend to turn from easy direction of the unit cell to the direction of the external magnetic field. Fig. 1, b shows how the unit cells of two variants are turned by external magnetic field. As the result, twins in preferred orientation to the magnetic field grow at the expense of the other twins. Ultimately, only one twin variant may remain, as shown in Fig. 1, c. The reorientation of the twin structure results in the shape change of the MSMA material. It is possible also to produce complex shape changes because the reorientation of the twin structure occurs in three dimensions.

![Fig. 1 Principle of magnetic-field-induced shape change of the twinned material](http://dx.doi.org/10.5755/j01.mech.18.3.1872)

2.2. Modeling theory

There are mainly following three kinds of models about the MSMA theory study:

1. Numerical value micro-magnetic model is suggested by James and Tickle [7, 8]; who proposed a model to calculate variant distribution, from which the strain and magnetization of the MSMA material can be solved. This model is based on minimization of the energy density in the MSMA material. The model is numerical so that it does not give analytical formulas for the magnetization and strain. Since it solves the exact variant distribution in the MSMA element, the demagnetization can be taken into account. Considering its detailed approach, the model should be the most accurate; however, the reported measured and calculated results differ from each other significantly. In addition, the lack of analytical result formulas reduces the usability of the model.

2. Analytic thermodynamic model is presented by
O’Handley [9]; who presented a model of the MSMA material in which the MSMA material element is studied as a bulk object, consisting of two variants. The basic idea of the model is to find the volume fractions of the variants of the material and then based on them calculate the magnetic field induced strain as well as the magnetization. Both magnetization and the macroscopic strain are assumed linearly dependent on the volume fractions. To solve the volume fractions, O’Handley formulated the equation for the free energy density in the material. This equation consists of magnetic energies, Zeeman energy and the magnetization energy of the reorientation variant as function of the rotation angle. Mechanically assuming the material is linear, the mechanical response to straining is taken into account with elastic energy. Later, the equation has been improved by the introduction of the external mechanical energy, which considers the external stress to the material [10]. A simplified hysteresis model between the magnetic field and strain was included in the main model [11, 12].

3. Thermodynamic model is proposed by Likhachev and Ullakko [13-15]. The model uses a principle, according to which both magnetic and mechanic forces will cause the same macroscopic deformation effects independent of the origin of the force [16]. The model explains the reason for the hysteresis between the magnetic field and strain being in the twinning stress of the material. Since there is a need for initial stress before the material gives significant strain, there is a critical field strength corresponding to the start of rapid growth, i.e. the switching magnetic field strength $H_{SW}$ [17]. This parameter depends on the strain value of the material as well as temperature.

2.3. Equivalent model of MSMA element

When the magnetic field is applied to MSMA element, it can output strain and stress. Hypothesis:

1. MSMA actuator is made up of the spring, the damper, the mass of a single freedom degree;

2. MSMA element has same length with coil; magnetic field strength $H$, magnetization $M$, strain $\varepsilon$ and stress $\sigma$ is uniform within MSMA element, element outputs the displacement $\Delta x = \varepsilon \times l_{MEMA}$, outputs force $F_0 = \sigma \times S_{MEMA}$, $l_{MEMA}$, $S_{MEMA}$ are the length and cross-sectional area of MSMA element respectively;

3. In the whole process of movement, the displacement of MSMA element is 0 at one end, and it is $\Delta x$ at the other end, the dynamics process of the MSMA element can be simplified as the equivalent mechanical model of single freedom degree, which is shown as in Fig. 2.

3. Output force and strain model of MSMA actuator

3.1. Experiment device

A specially designed experiment device is used for testing the static and dynamic characteristics of MSMA as shown in Fig. 3. The electromagnet is constructed of magnetically conductive core with air gap and excitation winding. The amplitude and direction of magnetic field in the air gap where the MSMA specimen is located can be controlled by the excitation current. The load force of MSMA specimen is set by the adjustment screw.

Four parameters, the strain, load force, magnetic field and temperature, need to be measured instantaneously for the characteristic test of the MSMA. The displacement of the MSMA specimen is transformed into DC voltage by an eddy-current sensor. The load force measurement is made by an electrical resistance strain gage with digital output. A linear Hall sensor is adopted for magnetic field measurement. The temperature is detected by means of a Pt thermo-resistance element with small size. The measured data for static test can be read from a digital displayer. However, the dynamic characteristics are recorded by a digital storage oscilloscope.

Fig. 3 Device schematic diagram of MSMA actuator

3.2. Model of output force under magnetic field

A material is called magnetically anisotropic if the magnetization curve of the material depends on the direction to which the material is magnetized. Some directions need only a little energy to be magnetized, while others need more. The direction that is easy to magnetize is called the easy magnetization direction (ED). The direction that needs most energy to be magnetized is called the hard magnetization direction (HD). Fig. 4 presents schematic views of the magnetization curves in HD and ED in the MSMA material.

The MSMA materials are ferromagnetic and anisotropic, which has a certain saturation value, and depends on the direction of the applied field [12, 13, 18-21]. The saturation value $B_s$ has ranged from 0.6 T [13] to 0.68 T [20] in different measurements. The saturation field strength $H_s$ falls within in the range 520 kA/m [21] to 720 kA/m [13]. The relative permeability has also varied between different measurements. The relative permeability at low magnetic field strength in the ED varies from 4.5 [13] to 100 [20] and in HD from 1.7 [13] to 2.2 [20]. Differences in these parameter values are most likely due to different compositions of MSMA and possibly different
annealing has a big influence especially on the permeability of the material [22]. In addition, the temperature has a significant influence on the magnetization curves of the material [23]. The anisotropy of the MSMA material is important for its operation. The higher the anisotropy energy, the higher the magnetic field induced stress. It has been shown that in MSMA martensite anisotropy grows as a function of the lattice distortion [24].

The expression of the easy and hard magnetization curve can be got from Fig. 5 respectively

\[ b_1(h) = \begin{cases} k_1 h + a_{12} & 0 < h < h_s \\ k_1 h - a_{12} & -h_s < h < 0 \end{cases} \]  \( (2) \)

\[ b_1(h) = k_1 h - a_{12} \quad -h_s < h < h_s \]

where \( h_s \) is the saturation magnetic field strength.

Eqs. (2) and (3) are substituted in Eq. (1), the magnetic field induced stress can be shown as

\[ \sigma_m(h) = \frac{1}{\varepsilon_0} \int_0^{h_s} \left[ b_1(h') - b_1 h' \right] dh \]

where \( b_1 \) and \( b_1 \) is the magnetic flux density along the easy and hard magnetization direction respectively, and \( h \) is the magnetic field strength.

To solve this equation, the magnetization curves must be measured along the easy and hard magnetization directions of the MSMA material. The measurement method of the magnetization curve is given by [19], which is shown as in Fig. 5.

The general thermodynamic properties of the MSMA materials which can show both the ferroelastic and the ferromagnetic properties, the ferroelastic reflects the mechanical properties through the relation of stress-strain in presence of magnetic field, the ferromagnetic gives the magnetization value as a function of magnetic field strength and strain. The ferroelastic and the ferromagnetic can be linked with Maxwell’s rule

\[ \frac{\partial}{\partial h} \sigma(\varepsilon, h) = -\frac{\partial}{\partial \varepsilon} m(\varepsilon, h) \]

Integration of this equation over the magnetic field starting from \( h = 0 \) at a fixed strain gives an important representation of the mechanical state equation including magnetic field effects

\[ \sigma(\varepsilon, h) = \sigma(\varepsilon, 0) - \frac{\partial}{\partial \varepsilon} \int_0^h d\varepsilon \left[ m(\varepsilon, h) \right] \]

The first term on the right is the pure mechanical stress resulting from the mechanical deformation of the material at \( h = 0 \) and the additional magnetic field induced stress that is represented by the second term on the right in this equation.

By deducing [13], the strain can be written as follows

\[ \varepsilon^n(h) = \frac{2}{3} \left( \frac{d\sigma_m(\varepsilon)}{d\varepsilon} \right)^{-1} \int_\varepsilon^0 dh \left[ m_s(h) - m_s(h) \right] \]

It can be seen from Eq. (7), the magnetic field-induced strain is connected with two factors:

1. the strain is proportional to the initial slope of stress-strain curve without magnetic field applied;
2. the integral term reflects the effects of magnetic anisotropy and determines the functional magnetic field dependence of the strain.
4. Analyze of output characteristics

4.1. Relationship between output force and magnetic field

MSMA element recovers the deformation by applying a magnetic field or an external force along the deformation direction. Actuator usually adopted the spring to realize the deformation recovery of MSMA material, the spring will apply a pre-pressure \( \sigma_0 \) to MSMA element, and the pre-pressure varies with the change of the material shape. The output force \( F_0 \) of MSMA element can be obtained from Fig. 2

\[
F_0 = (\sigma_n - \sigma_0) S_{\text{MSMA}} = (\sigma_n - k \Delta x) S_{\text{MSMA}} \quad (10)
\]

Because of the spring cannot completely recover the deformation of MSMA element, thus the dynamic strain of MSMA element is much less than a martensite lattice limit strain value. Ignore the pre-pressure variation caused by displacement change of MSMA material in Eq. (10), the pre-pressure can be regarded as a constant, the relationship between the output force and the magnetic field is shown as in Fig. 6 under the pre-pressure of 1 MPa, temperature of 26°C.

It can be seen from Fig. 6, the output force of MSMA element increases nonlinearly with respect to the increment of the magnetic field strength; the output force reaches the maximum value when the magnetic field achieves saturation value. On the other hand, the pre-pressure applying to the MSMA element can’t too high, otherwise the MSMA element cannot output force and displace.

Fig. 6 Relation between the output force and the field strength of the MSMA element

4.2. Relationship of output strain and magnetic field

When MSMA works as an actuator, the magnetic flux density of MSMA material is generally measured with Hall sensors, thus the magnetic field strength \( h \) can be replaced with the magnetic flux density \( b \) in Eq. (9). Considering the pre-pressure and temperature influence on the strain of MSMA material, then

\[
\varepsilon = k \frac{a_0 b^2 + a_1 b + a_2}{l_{\text{MSMA}}} \quad (11)
\]

Because actuator usually adopted the spring to realize the deformation recovery, the spring will apply a pre-stress \( \sigma_0 \) to MSMA element; when the field is excited by alternating current, copper loss and iron loss of the core can cause its temperature change, which can vary the strain of MSMA element. Considering the prestress \( \sigma_0 \) and temperature \( T \) influence on the strain of MSMA material, then

\[
\varepsilon = \frac{a_0 b^2 + a_1 b + a_2}{l_{\text{MSMA}}} (d_1 \sigma_0 + d_2) (c_1 T^2 + c_2 T + c_3) \quad (12)
\]

The every coefficient can be determined respectively by the experimental data [26] under the different magnetic field, pre-pressure and temperature.

Under the constant pre-pressure \( \sigma_0 \) (1 MPa) and temperature \( T \) (26°C), the calculations and experimental results between the strain and the magnetic flux density are shown in Fig. 7. The relation of the strain and the magnetic flux density is nonlinear. The increment of the strain tends to slow with the magnetic flux density strengthen, the calculated result and the experimental one is in good agreement.

Fig. 7 The curve of strain and magnetic flux density

It can be seen from Eq. (12), when the magnetic flux density reaches a certain value, continuing to increase the magnetic flux density, the magnetic field-induced strain which is determined by Eq. (12) will no longer increases with the magnetic field enhancing, instead of decreasing, and the experimental result shows that the magnetic field-induced strain remained constant because of the magnetic flux density reaches saturation. This is because the resistance of the twin boundary movement and the energy which needs to overcome the resistance also increases with the increment of the twin boundary motion quantity.
during the movement of the twin boundary. When the resistance of the twin boundary movement exceeds the energy of magnetic moment turning to the magnetic field in a way of rotating, magnetic moment will directly turn to the magnetic field by rotating. Zeeman energy difference between variants on both sides of the twin boundary tends to zero at the moment, the twin boundary no longer move, the magnetic field-induced strain reaches a constant value.

4.3. Relationship of output strain and temperature

Keeping the prestress and magnetic field constant ($\sigma_0 = 1$ MPa, $B = 0.51$ T), the calculations and experimental results between the strain and temperature are shown in Fig. 8. Temperature, the magnetic field and the external force can make MSMA element to produce deformation respectively, the deformation rate causing temperature is about 1%. When the temperature is close to the austenite transformation temperature ($A_t = 35.1^\circ$C), the strain reaches the maximum, after transformed into austenite, the strain is basically not appear.

![Fig. 8 The curve of deformation rate and temperature](image)

Because the movement of the martensite twin boundary depends on strongly temperature and the internal stress of the material, and temperature and the internal stress decide completely the energy barrier height, which need to overcome in the twin boundary movement, above the experiment result, the energy moving the twin boundary reflects the energy barrier height. Thus, the energy difference starting twin boundary movement is the reason that the magnetic field-induced strain depend on temperature under different temperature. The lower is temperature, the higher is the energy barrier hindering twin boundary movement, the variant occupying smaller volume fraction can only overcome the energy barrier and produce the re-orientation of short axis turning at the magnetic field, and the magnetic field-induced strain is smaller. Conversely, at higher temperature (such as nearly the phase transition temperature), the energy barrier hindering the twin boundary movement is smaller, so the magnetic field-induced strain is larger.

Obviously, at the same temperature, the magnetic field is applied along the crystal different directions, the difference of the energy for twin boundary moving and the twin boundary suffering the resistance is the importantly reason that the magnetic field-induced strain depends on temperature and the applied field direction. Therefore, the actuator should be kept the temperature constant or adopted temperature compensation as MSMA materials are used, and so as to eliminate the temperature impacting on the MSMA element deformation rate.

4.4. Relationship of output strain and pre-pressure

Keeping the magnetic flux density and the temperature constant ($B = 0.51$ T, $T = 26^\circ$C), the calculations and experimental results between the strain and pre-pressure are shown in Fig. 9.

![Fig. 9 The curve of deformation rate and pre-pressure](image)

Compressive stress is applied along the direction of MSMA element elongation, the variants of the short axis along the compressive stress direction gradually grow up by twin boundary movement, and the other variants, which of the short axis orientations are inconsistent with the compressive stress direction, are gradually “swallowed” and disappeared. The process of twin boundary movement corresponds to the reorientation of martensitic variants. Continuing to increase pressure, single crystal samples have approximately been pressed into single variant. The strain causing by the orientation of martensitic variants can not achieve the largest value of crystallographic strain. When the moving twin boundary comes across the upper or lower stress surface of the sample, the movement of twin boundary is hindered, the strain gradually decreases.

Increasing the stress to a higher value, intermediate martensitic transformation (the second reorientation) can be seen, it is the martensite secondary transformation, which can only produce a smaller strain; the stress is increased again, the variant arrangement of intermediate martensitic transformation is the end. During the dynamic process of the stress-induced intermediate martensitic transformation, any evidence of twin boundary movement is not observed. The second rearrangement of martensite variant is completed in manner of growing gradually of preferred orientation variant, and the first rearrangement of martensite variant is realized by twin boundary movement, which can gradually “swallowed” other unfavorable position variant.

5. Conclusion

An equivalent mechanical model is proposed on MSMA actuator. With the help of thermodynamic model and the calculation method of magnetic field-induced stress, the model of output strain and force are established when MSMA works as an actuator. The magnetic field-induced stress and strain in an MSMA material giving full 6% the magnetic field-induced strain are calculated
from magnetic anisotropy measurement, and the measured results are in accordance with the calculated values. The experiment results show that the model can calculate the output strain and force of the MSMA actuator under the magnetic field, pre-pressure and temperature.

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26. Fengxiang Wang; Wenjun Li; Qinxing Zhang; et al. 2006. Experimental study on characteristics of Ni-MnGa magnetically controlled shape memory alloy,
Magnetically controlled shape memory alloys (MSMA) offer a new way of producing motion and force, the most common usage of the materials are in the linear actuators. In order to design the MSMA actuators, modeling of the input dependence of the output for the actuators is necessary. Based on the calculation method of magnetic field-induced stress and the thermodynamic model, the models of output strain and force are presented for MSMA actuator in this paper. The model can quantitatively calculate the output strain and force dependence of the input magnetic field, temperature and stress. The output strain for a MSMA actuator is calculated, it is found that the calculating results are in a good agreement with the experimental ones, these indicate the validity and practicability of the actuator model.

**Keywords:** Magnetically controlled shape memory alloy, model, actuator, strain, stress.

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